
CHAPTER ONE

REVIEW OF ELECTROMAGNETIC PHENOMENON, VARIABLES AND CIRCUIT PARAMETERS

Electric charge

An Electric charge, denoted by Q is associated with a deficiency or the abundance of a group of electrons and is equal to the product of the number of electrons and the charge on each individual electron.

Electrical charge manifests itself in the form of forces--electrons repel other electrons but attract protons, while protons repel each other but attract electrons.

The unit of electrical charge is the coulomb(C). The **coulomb** is defined as the charge carried by 6.24 x 10^{18} electrons. Thus, if an electrically neutral (i.e., uncharged) body has 6.24 x 10^{18} electrons removed, it will be left with a net positive charge of 1 coulomb, i.e., Q = 1 C. Conversely, if an uncharged body has 6.24 x 10^{18} electrons added, it will have a net negative charge of 1 coulomb, i.e., Q = -1 C.

We can now determine the charge on one electron. It is Qe = 1/ (6.24 x 10^{18}) = 1.60 x 10^{-19} C.

Example - An initially neutral body has 1.7 μC of negative charge removed. Later, 18.7 x 10^{11} electrons are added. What is the body's final charge?

Solution - Initially the body is neutral, i.e., Q_{Initial} = 0 C. When 1.7 μC of electrons is removed, the body is left with a positive charge of 1.7 μC. Now, 18.7 x 10^{11} electrons are added back. This is equivalent to

$$18.7 \times 10^{11} electrons \times \frac{1\ coulomb}{6.24 \times 10^{18} electrons} = 0.3\mu C$$

of negative charge. The final charge on the body is therefore Q_f = 1.7 μC - 0.3 μC =1.4 μC.

Coulombs law

Coulomb determined experimentally that the force between two charges Q1 and Q2 is directly proportional to the product of their charges and inversely proportional to the square of the distance between them. Mathematically, Coulomb's law states

$$F = k\frac{Q_1 Q_2}{r^2} \quad newtons, N$$

Where Q_1 and Q_2 are the charges in coulombs, r is the center-to-center spacing between them in meters, and k= 9 x 10^9.

As Coulomb's law indicates, force decreases inversely as the square of distance; thus, if the distance between two charges is doubled, the force decreases to $(1/2)^2$ = 1/4 (i.e., one quarter) of its original value.

Negative result indicates attractive force and positive value indicates repulsive force.

Exercises:

1. Positive charges $Q_1 = 2\ \mu C$ and $Q_2 = 12\ \mu C$ are separated center to center by 10 mm. Compute the force between them. Is it attractive or repulsive?

2. Two equal charges are separated by 1 cm. If the force of repulsion between them is 9.7×10^{-2} N, what is their charge? What may the charges be, both positive, both negative, or one positive and one negative?

3. After 10.61×10^{13} electrons are added to a metal plate, it has a negative charge of 3 μC. What was its initial charge in coulombs?

Answers: 1. 2160 N, repulsive; 2. 32.8 nC, both (+) or both (-); 3. 14 μC (+)

<u>Electric Field</u>

By definition, the Electric field strength at a point is the force acting on a unit positive charge at that point; that is,

$$E = \frac{F}{Q} \quad (newton/coulomb, \qquad N/C)$$

The force exerted on a unit positive charge ($Q_2 = 1$ C), by a charge Q_1, r meters away, as determined by **Coulomb's law** is

$$F = \frac{kQ_1Q_2}{r^2} = \frac{kQ_1(1)}{r^2} = \frac{kQ_1}{r^2} \qquad (k = 9 \times 10^9\ Nm^2/C^2)$$
$$E = \frac{F}{Q_2} = \frac{kQ_1/r^2}{1}$$
$$E = \frac{kQ_1}{r^2} \quad (N/C)$$

<u>Voltage</u>

In electrical terms, a difference in potential energy is defined as **voltage.** In general, the amount of energy required to separate charges depends on the voltage developed and the amount of charge moved. By definition, *the voltage between two points is one volt if it requires one joule of energy to move one coulomb of charge from one point to the other*. In equation form,

$$V = \frac{W}{Q} \quad volts, \mathrm{V}$$

where W is energy in joules, Q is charge in coulombs, and V is the resulting voltage in voltsVoltage is defined between points. For the case of the battery, for example, voltage appears between its terminals. Thus, voltage does not exist at a point by itself; it is always determined with respect to some other point.

$$W = QV \quad joules, \mathrm{J}$$

$$Q = \frac{W}{V} \quad coulombs, \mathrm{C}$$

If we are dealing with a changing charge and energy, we have

$$\mathrm{v} = \frac{\mathrm{dw}}{\mathrm{dq}}$$

Example 2 - If it takes 35 J of energy to move a charge of 5 C from one point to another, what is the voltage between the two points?

Solution

$$V = \frac{W}{Q} = \frac{35\ \mathrm{J}}{5\ \mathrm{C}} = 7\ \mathrm{J/C} = 7\ \mathrm{V}$$

Exercices:

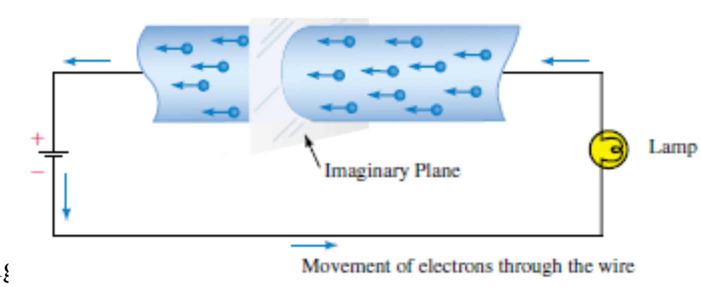
1. The voltage between two points is 19 V. How much energy is required to move 67 x 10^{18} electrons from one point to the other?

2. The potential difference between two points is 140 mV. If 280 μJ of work are required to move a charge Q from one point to the other, what is Q?

Answers: 1. 204 J 2. 2 mC

Current

Assume now that a battery is connected as in Figure 1.1. Since electrons are attracted by the positive pole of the battery and repelled by the negative pole, they move around the circuit, passing through the wire, the lamp, and the battery. This movement of charge is called an **electric current.** The more electrons per second that pass through the circuit, the greater the current. Thus, current is the *rate of flow* (or *rate of movement*) of charge.



Fi

Since charge is measured in coulombs, its rate of flow is coulombs per second. In the SI system, one coulomb per second is defined as one **ampere** (commonly abbreviated A). From this, we get that *one ampere is the current in a circuit when one coulomb of charge passes a given point (plane) in one second* (Figure 1.1). The symbol for current is I. Expressed mathematically,

$$I = \frac{Q}{t} \quad amperes, \mathrm{A}$$

where Q is the charge (in coulombs) and t is the time interval (in seconds) over which it is measured.

Alternate forms of the above equation are

$$Q = It \quad coulombs, \mathrm{C}$$

$$t = \frac{Q}{I} \quad seconds, \mathrm{s}$$

If we are dealing with time varying charge then we use the equation

$$\mathrm{i(t)} = \frac{\mathrm{dq(t)}}{\mathrm{dt}}$$

Then

$$\mathrm{q(t)} = \int_{\tau=-\infty}^{\mathrm{t}} \mathrm{i}(\tau)\mathrm{d}(\tau)$$

Example 3 – If 840 coulombs of charge pass through the imaginary plane of figure 1.1 during a time interval of 2 minutes, what is the current?

Solution - convert t to seconds

$$I = \frac{Q}{t} = \frac{840\ \mathrm{C}}{(2 \times 60)\mathrm{s}} = 7\,\mathrm{C/s} = 7\mathrm{A}$$

In the early days of electricity, it was believed that current was a movement of positive charge and that these charges moved around the circuit from the positive terminal of the battery to the negative as depicted in Figure 1–2(a). Based on this, all the laws, formulas, and symbols of circuit theory were developed. (We now refer to this direction as the *conventional current direction*.)

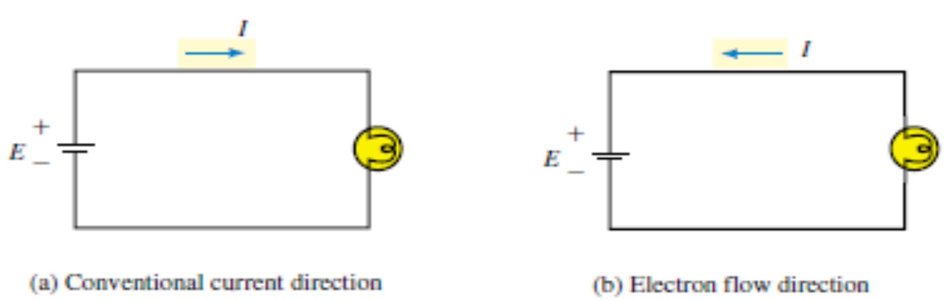


Figure 1.2 conventional and electron flow of electric current

After the discovery of the atomic nature of matter, it was learned that what actually moves in metallic conductors are electrons and that they move through the circuit as in Figure 1–2(b). This direction is called the *electron flow direction***.** However, because the conventional current direction was so well established, most users stayed with it. Thus, in most books and in the discussions that follow, the conventional direction for current is used.

Power

Power is defined as the rate of doing work or, equivalently, as the rate of transfer of energy. The symbol for power is *P*. By definition,

$$P = \frac{W}{t} \quad watts, \mathrm{W}$$

where *W* is the work (or energy) in joules and *t* is the corresponding time interval of *t* seconds.

The SI unit of power is the watt. From the above equation, we see that *P* also has units of joules per second. If you substitute *W* = 1 J and *t* = 1 s you get *P* = 1 J/1 s = 1 W. From this, you can see that *one watt equals one joule per second*.

To express *P* in terms of electrical quantities, recall that voltage is defined as work per unit charge and current as the rate of transfer of charge, i.e.,

$$V = \frac{W}{Q}$$

$$I = \frac{Q}{t}$$

From voltage equation, *W* = *QV*. Substituting this into Equation for power yields *P* = *W*/*t* = (*QV*)/*t* = *V* (*Q*/*t*). Replacing *Q*/*t* with *I*, we get

$$P = VI \quad watts, \mathrm{W}$$

Additional relationships are obtained by substituting *V* = *IR* and *I* =*V*/*R*

$$P = I^2R$$

$$P = \frac{V^2}{R}$$

For non constant (time varying) conditions

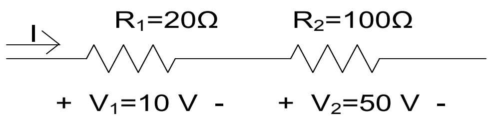
$$\mathrm{P(t)} = \frac{\mathrm{dW(t)}}{\mathrm{d(t)}}$$

$$\mathrm{P(t)} = \frac{\mathrm{dW(t)}}{\mathrm{dt}} = \frac{\mathrm{dW(t)}}{\mathrm{dq}} \times \frac{\mathrm{dq}}{\mathrm{dt}}$$

$$\mathrm{P(t)} = \mathrm{V(t)I(t)}$$

Example - compute the power to each resistor in figure below.

Solution –use the appropriate voltage in the power equation, for resistor R_1 use V_1 and for resistor R_2 use V_2



Solution

a. $P_1 = V_1^2/R_1 = (10\text{ V})^2/20\ \Omega = 5\text{ W}$

b. $P_2 = V_2^2/R_2 = (50\text{ V})^2/100\ \Omega = 25\text{ W}$

Exercises

a. show that $I = \sqrt{P/R}$ and $V = \sqrt{PR}$

b. A 100 Ω resistor dissipates 169 W. What is its current?

Energy

We defined power as the rate of doing work. When you rearrange this equation, you get the formula for energy:

$$W = Pt$$

If t is measured in seconds, W has units of watt-seconds (i.e., joules, J), while if t is measured in hours, W has units of watt-hours (Wh). Note that in the above equation, P must be constant over the time interval under consideration. If it is not, apply the equation to each interval over which P is constant.

For time varying circuits energy can be expressed as

$$w(t) = \int_{t=t_1}^{t_2} p(t)dt = \int_{t=t_1}^{t_2} v(t)i(t)dt$$

Example: Determine the total energy used by a 100 W lamp for 12 hours and a 1.5 kW heater for 45 minutes.

Solution - convert all quantities to the same set of units, thus 1.5 kW = 1500 W and 45 minutes = 0.75 h.then,

$$W = (100\ \mathrm{W})(12\ \mathrm{h}) + (1500\ \mathrm{W})(0.75\ \mathrm{h}) = 2325\ \mathrm{Wh} = 2.325\ \mathrm{kWh}$$

Example - Suppose you use the following electrical appliances: 1.5 kW heater for $7\,{}^{1}/_{2}$ hours, a 3.6 kW boiler for 17 minutes, three 100 W lamps for 4 hours, a 900 W toaster for 6 minutes. At 0.09 birr per kilowatthour, how much will this cost?

Solution - convert time in minutes to hours. Thus,

$$W = (1500)(7.5) + (3600)\left({}^{17}/_{60}\right) + (3)(100)(4) + (900)\left({}^{6}/_{60}\right)$$

$$= 13560\ \mathrm{Wh} = 13.56\ \mathrm{kWh}$$

$$\mathrm{cost} = (13.56\ \mathrm{kWh})(0.09\ \mathrm{birr/kWh}) = 1.22\ \mathrm{birr}$$

Faraday's law of Electromagnetic Induction

If a conductor is moved through a magnetic field so that it cuts magnetic lines of flux, a voltage will be induced across the conductor, as shown in Figure 1.3. The greater the number of flux lines cut per unit time (by increasing the speed with which the conductor passes through the field), or the stronger the magnetic field strength (for the same traversing speed), the greater will be the induced voltage across the conductor. If the conductor is held fixed and the magnetic field is moved so that its flux lines cut the conductor, the same effect will be produced.

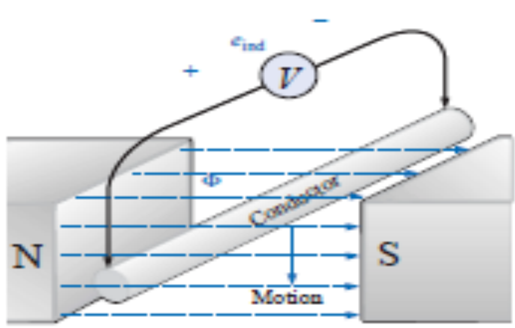


Figure 1.3 Induced voltage when a conductor wire passes through a magnetic flux

If a coil of *N* turns is placed in the region of a changing flux, a voltage will be induced across the coil as determined by Faraday's law:

$$e = N\frac{d\phi}{dt}\ \ \mathrm{volts, V}$$

Where e is voltage induced N represents the number of turns of the coil and $d\phi/dt$ is the instantaneous change in flux (in webers) linking the coil. The term *linking* refers to the flux within the turns of wire.

If the flux linking the coil ceases to change, such as when the coil simply sits still in a magnetic field of fixed strength, $d\phi/dt = 0$, and the induced voltage $e = N\,(d\phi/dt) = N(0) = 0$.

Self inductance

Self inductance of a coil is a measure of the change in flux linking a coil due to a change in current through the coil; that is,

$$L = N\frac{d\phi}{di}$$

Self Inductance can also be described as the measure opposition that an inductor exhibits to the change of current flowing through itself, measured in henrys (H). The opposition in the form of an induced voltage across the inductor is directly proportional to the time rate of change of the current.

The induced voltage is given by the formula

$$e_L = v = N\frac{d\phi}{dt} = (N\frac{d\phi}{di})(\frac{di}{dt})$$

$$e_L = L\frac{di}{dt}$$

where L is the constant of proportionality called the *inductance* of the inductor.

The inductance of an inductor depends on its physical dimension and construction. Inductors (coils) of different shapes have different formulas. Formulas for calculating the inductance of inductors of different shapes are derived from electromagnetic theory and can be found in standard electrical engineering handbooks.

Mutual inductance

In addition to inducing an opposing voltage in the original coil, change in current in a certain coil can also induce voltage across the terminals of another coil placed in its vicinity. This phenomenon is called mutual inductance.

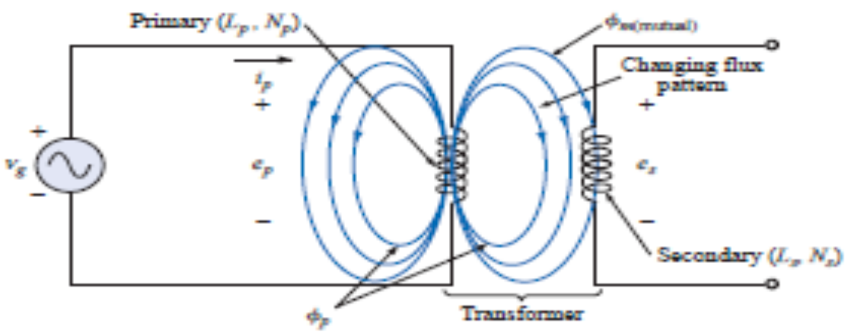


Figure 1.4 – mutual inductance in the primary and secondary coil of a transformer

Let us consider two coils, where the first one is called a primary coil (*p*) and the other one the secondary (*s*). The magnitude of e_s, the voltage induced across the secondary, is determined by

$$e_s = N_s \frac{d\phi_m}{dt}$$

where *Ns* is the number of turns in the secondary winding and ϕ_m is the portion of the primary flux ϕ_p that links the secondary winding.

If all of the flux linking the primary links the secondary, then $\phi_m = \phi_p$

$$e_s = N_s \frac{d\phi_p}{dt}$$

The mutual inductance between the two coils of the above figure is determined by

$$M = N_s \frac{d\phi_m}{di_p} \quad \text{henries, H}$$

$$M = N_p \frac{d\phi_p}{di_s} \quad \text{henries, H}$$

Circuit elements

Resistance and Resistors

A resistor is a material that provides an opposing force to the flow of charge through it. This opposition, due to the collisions between electrons and between electrons and other atoms in the material, which converts electrical energy into another form of energy such as heat, is called the **resistance** of the material. The unit of measurement of resistance is the **ohm,** for which the symbol is Ω (omega).

At a fixed temperature of 20°C (room temperature), the resistance is related to three factors by

$$R = \rho \frac{l}{A}$$

where ρ (Greek letter rho) is a characteristic of the material called the **resistivity,** l is the length of the sample, and A is the cross-sectional area of the sample.

The voltage- current relation of a resistor is determined by ohms law which is given by

$$I = \frac{V}{R} \quad \text{amperes, A}$$

where I is the current through the resistor and V is the voltage

Power

Power is an indication of how much work (the conversion of energy from one form to another) can be done in a specified amount of time, That is, a *rate* of doing work. In terms of the electrical quantities V and I,

$$P = \frac{W}{t} = \frac{QV}{t} = V\frac{Q}{t} \qquad but\ I = \frac{Q}{t}$$

$$P = VI \quad \text{watts, W}$$

By direct substitution of Ohm's law, the equation for power can be obtained in two other forms:

$$P = VI = V\left(\frac{V}{R}\right) = \frac{V^2}{R}$$

$$P = VI = (IR)I = I^2 R$$

Energy

The *energy* (*W*) lost or gained by any system is therefore determined by

$$W = Pt \quad \text{wattseconds, Ws, or joules}$$

The *wattsecond*, however, is too small a quantity for most practical purposes, so the *watthour* (Wh) and *kilowatthour* (kWh) were defined, as follows:

$$Energy(Wh) = power(W) \times time(h)$$

$$Energy(kWh) = \frac{power(W) \times time(h)}{1000}$$

Example – How much energy (in kilowatthours) is required to light a 60 W bulb continuously for 1 year?

Solution –

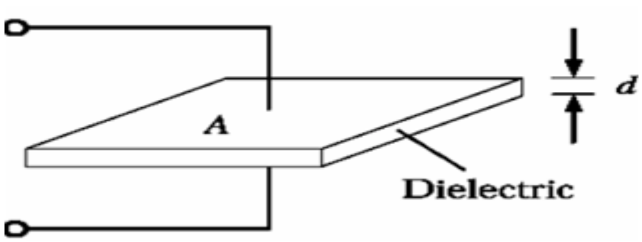
$$W = \frac{Pt}{1000} = \frac{(60\ W)(24\ h/day)(365\ days)}{1000} = \frac{525{,}600\ Wh}{1000}$$

$$= 520.60\ kWh$$

Exercise – How long can a 205 W television set be on before using more than 4 kWh of energy? *Ans* = 19.51 hours

Capacitance and Capacitor

A circuit element that is composed of two conducting plates or surfaces separated by a dielectric (non conducting) materials. If a voltage source (v) is connected to the capacitor, +ve charge will be transferred to one plate while –ve charge will be transferred to the other plate.



Let the charge stored at the capacitor $\equiv$q, then if v increases q also increases

$$v \propto q$$

Then from the above relation it can be found that

$$q = cv$$

Where c is the capacitance of the capacitor

$$c = \frac{q}{v}$$

The capacitance can also determined using the following relation

$$c \propto \frac{A}{d}$$

$$c \propto \frac{\varepsilon_o A}{d}$$

Where A=surface area of each plate

d=distance between the two plates

ε_o = Permittivity of free space

Current in capacitor

We know that

$$i(t) = \frac{dq(t)}{dt}$$

Then

$$i(t) = \frac{d}{dt}(cv_c(t))$$

$$i(t) = C\frac{dv_c(t)}{dt}$$

Voltage in capacitor

$$i_c(t) = C\frac{dv_c(t)}{dt}$$

$$i_c(t)dt = Cdv_c(t)$$

$$dv_c(t) = \frac{1}{c}i_c(t)dt$$

$$v_c(t) = v_c(t_o) + \frac{1}{c}\int_{\tau=t_o}^{\tau=t} i_c(\tau)d\tau$$

Where t_o is the initial time

The capacitor is a passive element and follows the passive sign convention

Inductance and Inductors

Inductors are circuit elements that consist of a conducting wire in the shape of a coil (N=1).

If a current is flowing in the inductor, it produce a magnetic field, Φ.

$$\Phi_c(t) = Li(t)$$

Where L is the inductance and measured in Henry [H]

As the current increases or decreases, the magnetic field spreads or collapse The change in magnetic field induces a voltage across the inductor.

$$v_L(t) = \frac{d\Phi(t)}{dt}$$

$$v_L(t) = L\frac{di_L(t)}{dt}$$

Current in inductors

$$v_L(t) = L\frac{di_L(t)}{dt}$$

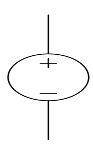
$$di_L(t) = \frac{1}{L}v_L(t)dt$$

Integrate both sides

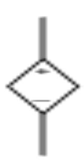
$$i_L(t) = i_L(t_o) + \frac{1}{L}\int_{\tau=t_o}^{\tau=t} v_L(\tau)\,d\tau$$

Electric sources

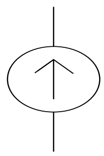
1. Independent voltage source: is a 2-terminal sources that maintains a specific voltage across its terminals regardless of the current through it. The circuit symbol of independent voltage sources is given below,



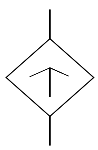
2. Dependent voltage source: is a 2-terminal sources that generates a voltage that is determined by a voltage or current at a specified location in the circuit. The circuit symbol of dependent voltage sources is,



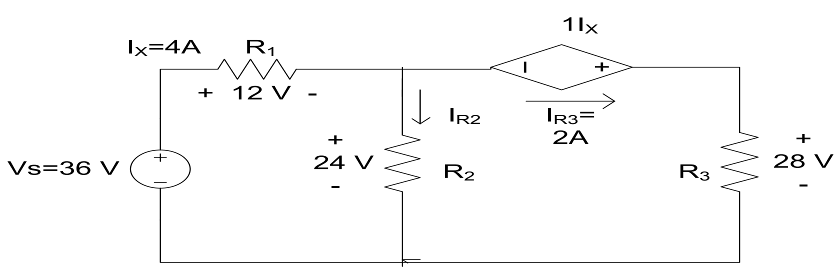
3. Independent current source: is a 2-terminal sources that maintains a specific current through it regardless of the voltage across it terminals. The circuit symbol of independent current sources is given below,



4. Dependent current source: is a 2-terminal sources that generates a current that is determined by voltage or current at a specified location in the circuit. The circuit symbol of dependent current sources is given below,



Example - Compute the power that is absorbed or supplied by each of the elements in the following circuit



Solution

$$P_{Vs} = V_s I_X = (36)(-4) = -144\ W \quad (supplies)$$

$$P_{R1} = V_{R1} I_X = (12)(4) = 48\ W \quad (absorbs)$$

$$P_{R2} = V_{R2} I_{R2} = V_{R2}(I_X - I_{R3}) = (24)(4-2) = 48\ W \quad (absorbs)$$

$$P_{Ds} = V_{Ds} I_{R3} = (1I_X)(I_{R3}) = (4)(-2) = -8\ W \quad (supplies)$$

$$P_{R3} = V_{R3} I_{R3} = (28)(2) = 56\ W \quad (absorbs)$$